CHEN 490 ~ Class 19 Th. 13/11/2014

Rotary Drilling Hydraulics

To rotary drilling operations, proper utilization of mud
Pump horsepower is important.

Analytical appraisal of the rig's circulating system must be understandable, specially the components which consume power, so that the available energy may be used correctly & advantageously.

The standard hydraulics approach to such analysis is hindered by many factors, among which are:

1. Mud flow property peculiarities. (discussed in mud composition, functions & rotary drilling fluids)
2. Irregularities of the circulating system

- Drilling mud leaves the pump discharge
- Passes thru the surface lines, stand pipe, and mud hose, and finally enters the drill string at the top of the kelly joint.
- Here, it begins the long downward travel thru the ~~nozzles~~ the drill pipe and drill collars, is expelled thru the nozzles of the bit, and return up the annulus.

The annular area is relatively small around the drill collars and becomes larger in the portion containing the drill pipe.

Since the mud enters the drill string and leaves the annulus at the same elevation, the only pressure required is that necessary to overcome the frictional losses in the system. Hence, the discharge pressure at the pump is defined by:

$$\Delta P_t = \Delta P_s + \Delta P_p + \Delta P_c + \Delta P_b + \Delta P_{ac} + \Delta P_{ap}$$

Where ΔP_t = pump discharge pressure

ΔP_s = pressure loss in surface piping, standpipe, and mud hose

ΔP_p = Pressure loss inside drill pipe

ΔP_c = Pressure loss inside drill collars

ΔP_b = Pressure loss across bit water courses or nozzles

ΔP_{ac} = pressure loss in annulus around drill collars

ΔP_{ap} = Pressure loss in annulus around drill pipe.

The solution of the above eqn. is rather tedious – seperate calculations for each section is required...

Before discussing plastic fluid flow calculations let us first review the fundamental equations of Newtonian fluid flow...

Required Flow Rate

Since mud is incompressible, the volume ~~or~~ or flow rate will be constant at any point in the circulating system. The flow velocity will very due to changes in cross-sectional area, or

$$q = Av = \text{constant (in consistent units)}$$

In the annulus, area A vary between the bore hole and drill collars, and tool joints, and drill pipe.

Due to the smaller diameter of the outside drill pipe, the area A adjacent to the drill pipe will be the largest, therefore the velocity will be the least or minimum.

To determine the required output flow rate of the mud pumps, an ~~the~~ experience ~~d~~ factor is used to determine the required minimum upward velocity in the annulus necessary for the efficient removal of the cuttings. This "experience" velocity is given in ft/min and can be used to calculate the required flow rate, or

$$q = Av \text{, or}$$

$$q = 2.45\,(d_{hole}^2 - d_{OD\,pipe}^2)\,v$$

where q = gal/min

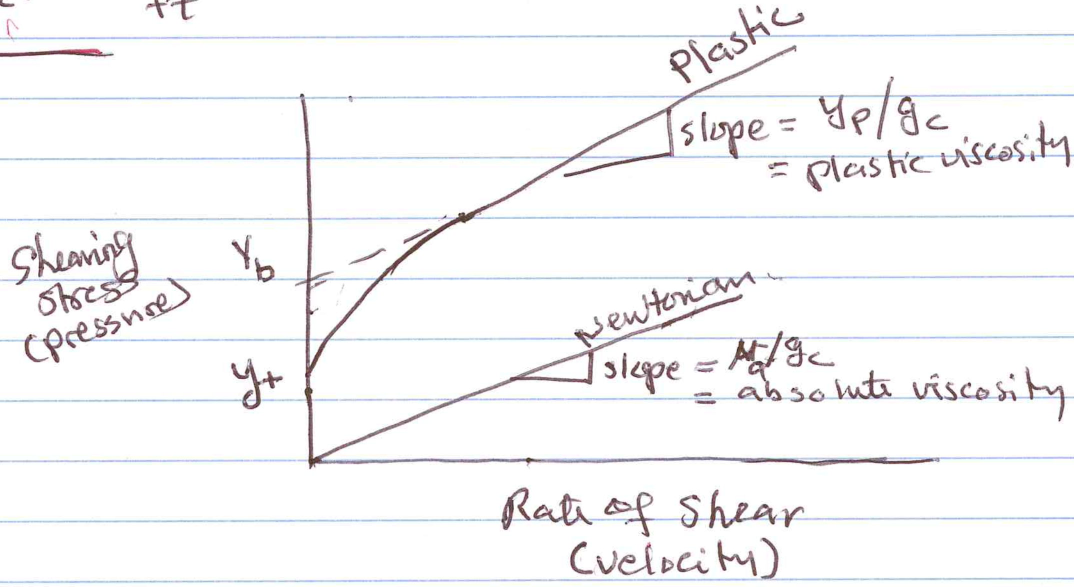
d = in , v = ft/sec

Viscosity:

Viscosity of a fluid is defined as the ratio of the shearing stress to the rate of shear, a

$\frac{\text{Force}}{\text{Area}} / \frac{\text{Velocity}}{\text{Distance}}$; pressure/Rate of velocity change

$\left(\frac{lb_f}{ft^2} / \frac{(ft/sec)(g_c)}{ft} = lb_m/ft\text{-}sec$



Referring to the above figure;

A Newtonian fluid is defined as a fluid having the rate of shear ~~velocity~~ directly proportional to the applied shearing stress or a straight line thru the origin, ∴ μ_a is constant.

A plastic fluid is defined as a fluid which requires a definite shearing stress be applied before any shearing occurs. μ_p is not constant until a certain value of shearing stress is applied.

To determine the viscosity of a Newtonian fluid, it is necessary to determine only one pt., then a line thru ~~the origin~~ the point and the origin establishes the slope or μ_a/g_c

Determining the viscosity of a plastic fluid requires enough points to establish the straight line portion of the curve.

Each plastic fluid will have its own true yield point, Y_t [which is the value of shearing stress required to produce movement of the fluid]. Increasing the shear stress above Y_t produces a transition zone in which μ_p is not constant.

Depending on the fluid, the points will become linear or a constant slope $= \mu_p / g_c$. Extending the straight line portion of the plastic fluid, it is necessary to have both the slope of the straight line portion $= \mu_p / g_c$ and either the true or the Bingham yield point, Y_t or Y_b. [It can be shown that the Bingham yield point equals 4/3 the true yield for all plastic fluids,] or

$$Y_b = \frac{4}{3} Y_t$$

Types of Flow:

Laminar flow occurs when all individual particles in the fluid flow in a straight line parallel to the axis of the conductor. Under certain conditions (velocity, viscosity, density, and diameter of the conductor),

Turbulent flow occurs when the particles flow in a random manner.

The Reynolds Number, N_R, relationship is used to determine type of flow under given conditions, or

$$N_R = \frac{d \rho v}{\mu} \quad , \ \rho,\ \text{lbm/ft}^3,\ v\ (\text{basic units})$$

when N_R = dimensionless number

d = diameter, ft

v = velocity, ft/sec

μ = viscosity, lbm/ft-sec

ρ = density, lbm/ft^3

or

$$N_R = \frac{(928)\, d \rho v}{\mu} \quad (\text{field units})$$

Thru experiment, laminar flow exists when the value of the Reynolds Number is less than 2000 and turbulent flow exists when the value of Reynolds Number is greater than 2000. or

~~Newtonian Fluid Laminar Flow in Pipe~~

$N_R < 2000$; Laminar flow

$N_R > 2000$; Turbulent flow

Newtonian Fluid Flow Calculations

The Hagen-Poiseuille eqn. states the relationship between Pressure drop due to friction and other flow factors for a

Newtonian fluid under laminar flow conditions in a straight circular pipe (for instance), or

$$\boxed{\Delta P_f = \frac{32 \mu L v}{g_c d^2} \text{ (Basic units)}}$$

or

$$\boxed{\Delta P_f = \frac{\mu L v}{1500 d^2} \text{ (Field units)}}$$

where, ΔP_f = psi, L = ft, μ = cp, v = ft/sec, d = in

For Turbulent Flow —

The Fannin Equation states the relationship between pressure drop due to friction and other flow factors for a Newtonian fluid under turbulent flow conditions in a straight, circular pipe (for instance), or

$$\Delta P_f = \frac{2 f \rho L v^2}{g_c d} \text{ (Basic units)}$$

or

$$\boxed{\Delta P_f = \frac{f \rho L v^2}{25.8\, d} \text{ (Field units)}}$$

where f = Fannin Friction Factor, f = dimensionless

f is a function of N_R and has been evaluated ~~under~~ experimentally for numerous materials and has been related to diameter of the conductor and the condition of the surface of the inside of the conductor.

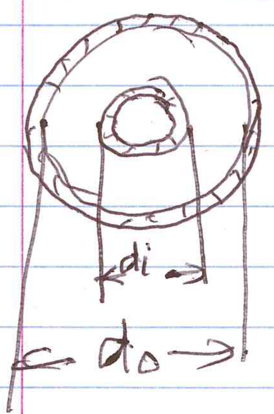
Newtonian Fluid – Laminar flow – In annulus

The Poiseuille eqn. for laminar flow applies only to a straight, circular pipe and can not be used if the area is an annulus. To use this eqn. for an annulus, the annular area must be expressed as an equivalent area of a pipe which will have the same pressure drop per length at the same flow rate.

diameter of the annulus, or equivalent diameter, d_e, is defined as 4 times the hydraulic radius, and the hydraulic radius is defined as,

$$r_h = \frac{\text{Cross sectional area of flow}}{\text{wetted parameter}}$$

where wetted perimeter is the total length of surface contacted by the fluid, or



d_o = inside diameter of the outside conductor

d_i = outside diameter of the inside conductor

$$\text{so, } d_e = 4\, r_h = \frac{(4\pi/4)(d_o^2 - d_i^2)}{\pi(d_o + d_i)} = d_o - d_i$$

the actual velocity, v_a, in an annulus is

$$v_a = \frac{q}{2.45(d_o^2 - d_i^2)} \quad \text{ft/sec}$$

Type of Flow in Annulus

Calculate the equivalent Reynolds Number using the equivalent diameter and the actual velocity, or

$$N_{re} = \frac{757\,(d_o - d_i)\,\rho\,\bar{v}_a}{\mu};\quad N_{Re} < 2000,\ \text{laminar flow}$$

$$N_{re} = \frac{757\,(d_e)\,\rho\,\bar{v}_a}{\mu};\quad N_{Re} > 2000,\ \text{turbulent flow}$$

If laminar, use the Poiseuille equation modified for annular flow to calculate pressure drop, or

$$\Delta P_f = \frac{\mu L \bar{v}_a}{1000\,(d_o - d_i)^2} = \frac{\mu L \bar{v}_a}{1000\,(d_e)^2}\quad (\text{field units})$$

Newtonian fluid – Turbulent flow – in Annulus

The Fannin eqn. for turbulent flow applies. Use also equivalent diameter of the annulus

$$d_e = (d_o - d_i)$$

Type of Flow in an Annulus:

Calculate the equivalent Reynolds Number using the equivalent diameter and the actual velocity, or

$$N_{re} = \frac{757\, d_e\, \rho\, \bar{v}_a}{\mu};\quad N_{Re} < 2000,\ \text{laminar};\quad N_{Re} > 2000,\ \text{turbulent}$$

If turbulent flow, use the Fannin equation to calculate pressure drop, or

$$\Delta P_f = \frac{f \rho L \bar{v}_a^2}{25.8\, d_e}$$

where f = fannin friction factor; dimensionless
ρ = lbm/gal, L = ft, v = ft/sec, d = in

~~f~~ The fannin friction factor, f, has been related by experiment to the Reynolds Number, N_R, diameter of the conductor, & the condition of the surface of the inside ~~o~~ of the conductor.

Friction factor, f, can be shown as a function of ~~R~~ N_R, & relative Roughness, ε/d. The value of ε is 0.00065 in.

The following friction factor curve have been given ~~as a function~~ shows the ~~of N_R~~ relation between f, N_R for ~~so~~ various conditions found in pipes & annular spaces.

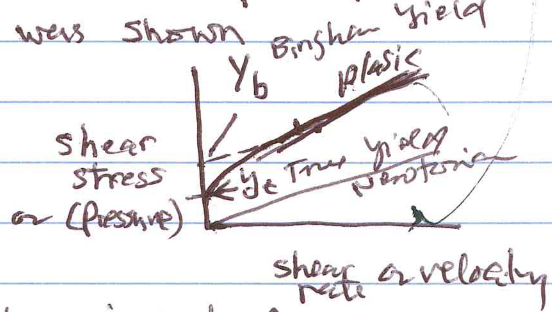
The N_R is calculated first to determine
∴ If flow is laminar or turbulent.
If turbulent, use the friction factor curves to determine f which will be used in the Fannin Eqn.

Plastic Fluid - Laminar flow - In Pipe

Laminar Flow Region

[The plastic viscosity curve, the eqn. ~~of~~ for the ~~plastic~~ straight line portion of the curve is]

The typical pressure - velocity behavior of a plastic fluid flowing thru a pipe was shown



A definite pressure (Y_t) is required to initiate flow. True laminar flow is represented by the linear portion of the curve, the eqn. of which is

$$144\Delta p = \frac{4}{3}Y_t + mv$$

Where $144\,\Delta p$ = pressure drop (Δp_f), lb/ft²

$\frac{4}{3}Y_t = Y_b$, the Bingham yield value, lb/ft²

m = slope of linear portion, which is proportional to the plastic viscosity, μ_p. Note: $m = \frac{\mu L}{1500 d^2}$, from the poiselle's eqn.

$$\Delta p = \frac{\mu L v}{1500 d^2}$$

[i.e. $m = \frac{\Delta p_f}{v}$]

For practical values of $\bar{v}$, the behavior of Bingham fluids may be expressed as:

$$\Delta p_f = \frac{L Y_b}{300 d} + \frac{\mu_p \bar{v} L}{1500 d^2} = \frac{L}{300 d}\left(Y_b + \frac{\mu_p \bar{v}}{5d}\right)$$

Where Y_b = yield point, lb/100 ft², μ_p = plastic viscosity, cp, or

$$\Delta P_f = \frac{\mu_p L v}{500 d^2} + \frac{Y_b L}{300 d}$$

The Reynolds Number eqn. and Hagen-Poiseille eqns for laminar flow apply only to a Newtonian fluid flowing in a straight, circular pipe. If these eqns. are to be used for a plastic fluid, an equivalent viscosity, μ_e which is the viscosity a plastic fluid would have if it were a Newtonian fluid, must be used, and,

$$\mu_e = \frac{5 Y_b d}{v} + \mu_p$$

Since μ_e is an equivalent Newtonian viscosity, it can be used in Reynolds Number eqn, or

$$N_R = \frac{928 d e v}{\frac{5 Y_b d}{v} + \mu_p} \quad ; \quad N_R < 2000 \text{ laminar flow}, \quad N_R > 2000, \text{ turbulent flow}$$

Setting $N_r = 2000$ and solving for velocity yields a critical velocity, v_c, and an actual velocity below which is laminar flow and an actual velocity above which is turbulent flow, or

$$v_{ac} < v_c \text{ — laminar flow}$$
$$v_{act} > v_c \text{ — turbulen}$$

then,

$$N_R = 2000 = \frac{928 d e v}{\frac{5 Y_b d}{v} + \mu_p}$$

or

$$v_c = \frac{1.08 \mu_p + 1.08 (\mu_p^2 + 9.3 e d^2 Y_b)^{0.5}}{e d}$$

and

$$v_{act} = \frac{q}{2.45d^2}$$

OR

$$\Delta p = \frac{L Y_b}{300d} + \frac{\mu_p \bar{v} L}{1500d^2} = \frac{L}{300d}\left(Y_b + \frac{\mu_p \bar{v}}{5d}\right)$$

Critical velocity — Calculation

Equate the above eqn. to $\Delta p = \frac{\mu L v}{1500d^2}$, an equivalent Newtonian viscosity in terms of $d, \bar{v}, \mu_p$, and Y_b is obtained

$$\mu = \frac{5d Y_b}{\bar{v}} + \mu_p$$

substituting this eqn. for μ in the Reynold's Number eqn., equating the resulting eqn. to 2000, and solve for v gives

$$V_c = \frac{1.08\mu_p + 1.08\sqrt{\mu_p^2 + 9.3\rho d^2 Y_b}}{\rho d}$$

where

critical velocity, ft/sec, above which turbulent flow exists and below which the flow is laminar.

The eqn. assumes that turbulence occurs at $N_{Re} = 2000$, i.e., $\bar{v} < v_c$, flow is laminar

$\bar{v} > v_c$ flow is turbulent.

Turbulent Flow Calculations

Fannin eqn. may be used for turbulent flow calculations — providing the Reynolds number expression is altered by substitution of

$$\mu_t = \frac{\mu_p}{3.2} = \text{turbulent viscosity of plastic fluid}$$

Substituting this μ_t for μ in the general Reynolds number eqn. gives

$$N_R = \frac{928 \rho \bar{v} d}{\mu_t} = \frac{2970 \rho \bar{v} d}{\mu_p}$$

Plastic Fluid - Laminar flow - In annulus:

The Hagen-Poiseuille eqn. for laminar flow applies only to a straight, circular pipe and cannot be used directly ~~to~~ if the cross-section area is an annulus.

To use this eqn. for an annulus, the area must be expressed as an equivalent area of pipe which will have the same pressure drop per length at the same flow rate. This expression is the diameter of of a pipe which will have the same Δp - flow rate relation at the equivalent diameter of the Annulus, or

equivalent diameter, d_e, is defined as,

$$r_h = \frac{\text{Cross-sectional area of flow}}{\text{Wetted parameter}}$$

where, the wetted parameter is the total length of surface connected by the fluid,

d_o = inside diameter of the outside conductor

d_i = outside diameter of the inside conductor

$$d_e = \frac{(4)(\pi/4)(d_o^2 - d_i^2)}{\pi(d_o + d_i)} = \underline{d_o - d_i}$$

Type of Flow in an Annulus:-

The expression for determining the critical velocity, v_c is modified when the flow is in annulus, or

$$V_c = \frac{1.08\mu_p + 1.08(\mu_p^2 + 6.98\rho d_e^2 Y_a)^{0.5}}{\rho\, d_e}$$

$d_e = d_o - d_i$

Determine if the flow is laminar or turbulent by calculating the critical velocity, V_c, using $(d_o - d_i)$ and comparing to the actual velocity, V_{act}.

$V_{act} < V_c \longrightarrow$ laminar flow

$V_{act} > V_c \longrightarrow$ turbulent flow

If laminar flow, the Hagen-Poiseuille eqn. modified for annular flow must be used to calculate ΔP using d_e, V_{act} and μ_p

$$\Delta P_f = \frac{\mu_p L V_{act}}{1000(d_o - d_i)^2} + \frac{Y_B L}{267(d_o - d_i)}$$

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ON MY HONOR, I WILL NOT GIVE OR RECEIVE
ANY ASSISTANCE ON THIS QUIZ OR EXAM.

Signature: ____________________

Type of flow

Laminar flow
Turbulent flow

The Reynolds Number, N_R, relationship is used

$$N_R = \frac{(928)(d)(\rho)v}{\mu}$$

Where
N_R = dimensionless number
d = diameter, in
ρ = density, lbm/gal
v = velocity, ft/sec
μ = viscosity, cp

Thru experiment

$N_R < 2000 \Rightarrow$ laminar flow
$N_R > 2000 \Rightarrow$ Turbulent flow

Newtonian Fluid - Laminar flow - In Pipe

Poiseuille equation states the relationship between Pressure due to friction and other flow factors for a Newtonian fluid under laminar flow conditions in a straight, circular pipe, or

$$\Delta P_f = \frac{\mu L v}{1500\, d^2}$$

Where ΔP_f = psi, L = ft, d = in, μ = cp, v = ft/sec

d = in

Newtonian fluid - Turbulent flow - In pipe

The Fannin equation states the relationships between pressure drop due to friction and other flow factors for a Newtonian fluid under turbulent flow conditions in a straight, circular pipe, or

$$\Delta P_f = \frac{f \rho L v^2}{25.8\, d}$$

Where f = dimensionless = fannin friction Factor related by experiment to the Reynolds Number, N_R, diameter of the conductor, and the condition of the surface of the inside of the conductor. Can be shown as a function of N_R.

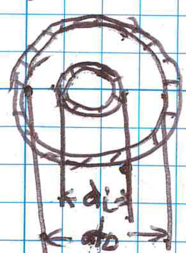
Newtonian fluid - laminar flow - In Annulus

To use the Poiseuille eqn. for laminar in the annulus, the annulus cross-section area must be expressed as an equivalent cross-sectional area of a pipe which will have the same pressure drop / length of the same flow rate.

Equivalent diameter of the annulus, de, is defined as 4 times the hydraulic radius, and the hydraulic radius is defined as

$$r_h = \frac{\text{Cross-sectional area of flow}}{\text{Wetted Parameter}}$$

Where, the wetted perimeter is the total length of surface contacted by the field or



d_o = inside diameter of the outside conductor
d_i = outside diameter of the inside conductor

$\therefore$ $$d_e = 4r_h = \frac{(4)(\pi/4)(d_o^2 - d_i^2)}{\pi(d_o + d_i)} = (d_o - d_i)$$

the actual velocity, v_{act}, in an annulus is,

$$v_{act} = \frac{q}{2.45(d_o^2 - d_i^2)} \quad ft/sec$$

<u>Type of flow in an annulus</u>

Calculate the equivalent Reynolds Number using the equivalent diameter and the actual velocity or

$$N_{re} = \frac{757(d_o - d_i)\rho v_{act}}{\mu} \Rightarrow N_{re} < 2000, \text{ laminar flow}$$

$$N_{re} = \frac{757\rho d_e v_{act}}{\mu} \Rightarrow N_{re} > 2000, \text{ turbulent flow}$$

If laminar use the Poiseulle equation modified for annular flow to calculate pressure drop, or

$$\Delta P_f = \frac{\mu L v_{act}}{1000(d_o - d_i)^2}$$

<u>Newtonian fluid - Turbulent flow - in Annulus</u>

$$d_e = d_o - d_i$$

<u>Type of flow in an annulus</u>

$$N_{re} = \frac{757 d_e \rho v_{act}}{\mu} \Rightarrow N_{re} < 2000 \Rightarrow \text{laminar flow}$$

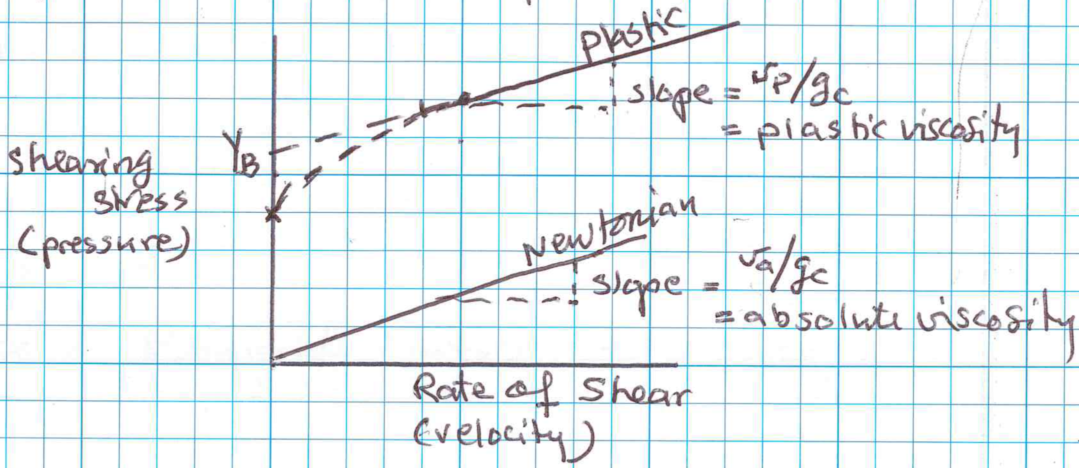
$$N_{re} > 2000 \Rightarrow \text{turbulent flow}$$

If turbulent, use the Fannin equation to calculate pressure drop

$$\Delta P_f = \frac{f L v_{act}^2}{25.8 d_e}$$

Plastic Fluid-Laminar flow - in Pipe

Referring to the plastic fluid viscosity curve shown below



The eqn. for the straight line portion of the curve is

$$\Delta P_f = m v + Y_B$$

and the Poiseuille eqn. for laminar flow is,

$$\Delta P_f = \frac{32 \mu v L}{g_c d^2}$$

or

$$\text{Slope} = m = \frac{\Delta P_f}{v} = \frac{32 \mu L}{g_c d^2}$$

then,

$$\Delta P_f = \frac{(32 \mu L)(v)}{g_c d^2} + Y_B$$

and

Y_B expressed in equivalent pressure terms $= \dfrac{4 Y_B L}{d}$

then

$$\Delta P_f = \frac{32 \mu_p L v}{g_c d^2} + \frac{4 Y_B L}{d}$$

or

$$\Delta P_f = \frac{\mu_p L v}{500 d^2} + \frac{Y_B L}{300 d} \quad \text{(Field units)}$$

If these equations are to be used for a plastic fluid, an equivalent viscosity, μ_e, which is the viscosity a plastic fluid would have if it were a Newtonian fluid, must be used, and

$$\mu_e = \frac{5Y_B d}{v} + \mu_p$$

Since μ_e is an equivalent Newtonian viscosity, it can be used in Reynolds Number equation,

or,

$$N_R = \frac{928\, d \rho v}{\frac{5Y_B d}{v} + \mu_p}$$; $N_R < 2000$, laminar ; $N_R > 2000$, turbulent

Since $N_R = 2000$ and solving for velocity yields a critical velocity, v_c, and an actual velocity below which is laminar flow and an actual velocity above which is turbulent flow,

or $v_{act} < v_c$, laminar flow

$v_{act} > v_c$, turbulent flow